Coupled Distributed Estimation and Control for Mobile Sensor Networks

Reza Olfati-Saber and Parisa Jalalkamali

Abstract—In this paper, we introduce a theoretical framework for coupled distributed estimation and motion control of mobile sensor networks for collaborative target tracking. We use a Fisher Information theoretic metric for quality of sensed data. The mobile sensing agents seek to improve the information value of their sensed data while maintaining a safe-distance from other neighboring agents (i.e. perform information-driven flocking).

We provide a formal stability analysis of continuous Kalman-Consensus filtering (KCF) algorithm on a mobile sensor network with a flocking-based mobility control model. The discrete-time counterpart of this coupled estimation and control algorithm is successfully applied to tracking of two types of targets with stochastic linear and nonlinear dynamics.

Index Terms—mobile sensor networks, distributed Kalman filtering, flocking, information-driven control, collaborative target tracking

I. INTRODUCTION

Collaborative tracking of multiple targets (or events) in an environment arise in a variety of surveillance and security applications and intelligent transportation. Most of the past research on target tracking has been focused on the use of centralized algorithms that run on static multi-sensor platforms [1]. Centralized Kalman filtering plays a crucial role in such target tracking algorithms.

Distributed estimation for static sensor networks has attracted many researchers in recent years [2], [3], [4], [5], [6], [7]. The existing distributed algorithms for target tracking using mobile sensor networks are extremely limited to a few instances [8], [9]. In [10] the KCF algorithm of the first author is successfully applied to multi-target tracking using camera networks.

In this paper, we present a systematic analysis framework for mobile sensor networks with a flocking-based mobility control model that run a novel distributed Kalman filtering algorithm [11] for collaborative tracking of a single target.

The sensors in our framework have an information value function \( I_i = f(\rho_i) \) where \( \rho_i \) denotes the target range and defined as the distance between the agent and the predicted position of target \( \gamma \). In addition, \( f(\rho) \) is a decreasing function of the target range. According to this model of quality of sensed data, the information value of a sensor increases as the sensor comes closer to the target. This notion of the information value that was also used in [8] is the same as the trace of the Fisher Information Matrix (FIM) of sensed data for target tracking applications [12], [13].

We propose a solution to the problem of collision-free tracking of a mobile target via mobile sensor networks using a combination of the flocking and Kalman-Consensus Filtering algorithms [2], [11] of the first author.

The major challenge in analysis of the resulting coupled estimation and control algorithm for mobile sensor networks that we call information-driven flocking is that each sensing agent \( \alpha_i \) has its own dedicated \( \gamma \)-agent called \( \gamma_i \) (See [14] for the definition of \( \alpha \)- and \( \gamma \)-agent). The state of \( \gamma_i \) is the estimate of the state of target \( \gamma \) by agent \( i \) and the \( n \) different estimates \( \gamma_i \) of the target are distinct. In the flocking algorithms presented in [14], all \( n \) \( \gamma \)-agents are the same. This change results in a perturbed structural dynamics of the flock where the perturbation terms depend on the estimation errors.

Our main result is to establish that the coupled distributed estimation and control algorithm for a mobile sensor network has a combined cost (Lyapunov function) that is monotonically decreasing in time and guarantees reaching a consensus on estimates of the state of the target by all mobile sensors. We also introduce a cascade nonlinear normal form and stability analysis for structural dynamics of mobile sensor networks performing information-driven flocking.

The outline of the paper is as follows. Some basic notations and problem setup are discussed in Section II. Our main theoretical results on distributed target tracking algorithms for mobile sensor networks are provided in Section III. Our experimental results are presented in Section V. Finally, concluding remarks are made in Section VI.

II. PRELIMINARIES: NOTATIONS AND PROBLEM SETUP

Consider \( n \) mobile sensors \( \alpha_i \) with the dynamics

\[
\begin{aligned}
\dot{q}_i &= p_i \\
\dot{p}_i &= u_i
\end{aligned}
\]  (1)

where \( q_i, p_i, u_i \in \mathbb{R}^d \) and the goal to track the state of a mobile target \( \gamma \) with dynamics

\[ \dot{x} = Ax + Bw; \quad x \in \mathbb{R}^m \]  (2)

The sensing agents make the following partial-state noisy measurements of the state of \( \gamma \)

\[ z_i = H_i x + v_i, \quad i = 1, 2, \ldots, n; z_i \in \mathbb{R}^l \]  (3)

where the matrices \( A, B, \) and \( H_i \) are generally time-varying and of appropriate dimensions and \( w \) and \( v_i \) are zero-mean Gaussian noise.

R. Olfati-Saber is an Assistant Professor of Engineering at the Thayer School of Engineering at Dartmouth College, Hanover, NH. E-mail: olfati@dartmouth.edu. This work was supported in part by the NSF CAREER award of the author.

P. Jalalkamali is a PhD candidate at the Thayer School of Engineering at Dartmouth College, Hanover, NH. E-mail: parisa.jalalkamali@dartmouth.edu.
Let $G = G(q)$ be the proximity graph (network) of the mobile sensors. The set of vertices of $G$ is $V = \{1, 2, \ldots, n\}$. Let $r > 0$ be the interaction range of every sensor. Then, the set of edges of $G$ is a time-varying set defined as

$$E(q) = \{(i, j) \in E : \|q_j - q_i\| < r\}$$  \hspace{1cm} (4)

and the set of neighbors $N_i$ of sensor $i$ on this proximity network is given by

$$N_i = \{j \in V : \|q_j - q_i\| < r\}.$$

The main problem of interest is to design distributed motion control and estimation algorithms that achieve two objectives: i) the group of sensing agents improve their collective information value $\sum_i I_i$ and ii) avoid collisions during tracking of target $\gamma$. We refer to this problem as “information-driven flocking.” We propose a solution to this problem using a combination of flocking and Kalman-Consensus Filtering algorithms [11].

### III. DISTRIBUTED TRACKING WITH MOBILE SENSORS

The Kalman-Consensus algorithm (or Algorithm 1) relies on reaching a consensus on estimates obtained by local Kalman filters rather than distributed averaging-based Kalman filtering. Algorithm 1 is the discrete-time analog of the continuous-time Kalman-Consensus filter described in the following.

**Theorem 1. (Kalman-Consensus Filter [21])** Consider a sensor network with a continuous-time linear sensing model in (3). Suppose each node applies the following distributed estimation algorithm

$$\dot{x}_i = A\dot{x}_i + K_i(z_i - H_i\dot{x}_i) + \mu P_i \sum_{j \in N_i} (\dot{x}_j - \dot{x}_i)$$  \hspace{1cm} (5)

with a Kalman-Consensus estimator and initial conditions $P_i(0) = P_0$ and $\dot{x}_i(0) = x(0)$. Then, the collective dynamics of the estimation errors $\eta_i = x - \dot{x}_i$ (without noise) is a stable linear system with a Lyapunov function

$$V(\eta_i) = \sum_{i=1}^n \eta_i^T P_i^{-1} \eta_i.$$  \hspace{1cm} (6)

Moreover, $\dot{V} \leq -2\mu V_i(\eta_i) \leq 0$ where

$$V_i(\eta_i) = \dot{x}_i^T L \dot{x}_i = \frac{1}{2} \sum_{(i, j) \in E} \|\dot{x}_j - \dot{x}_i\|^2$$

and $L = \sum_{i=1}^n \eta_i^T P_i^{-1} \eta_i \sum_{i=1}^n \eta_i$ is the $m$-dimensional Laplacian of the network ($\otimes$ denotes the Kronecker product). Moreover, all estimators asymptotically reach a consensus, i.e. $\dot{x}_i = x, \forall i$.

The following flocking algorithm is a modified form of Algorithm 2 in [14].

**Algorithm 2:** (flocking with $n$ distinct $\gamma$-agents) Let $\hat{x}_i = \text{col}(\hat{q}_i, \dot{\hat{q}}_i, p_v, \dot{p}_v)$ be the estimate of the state of target $\gamma$ by mobile sensor $i$, obtained via Kalman-Consensus filtering. Then, each sensing agent $a_i$ with dynamics in (1) applies the following distributed control to interact with its neighboring sensors on $G(q)$:

$$u_i = \sum_{j \in N_i} \phi_a(\|q_j - q_i\|)n_{ij} + \sum_{j \in N_i} \phi_a(\|q_j - q_i\|)n_{ij} + a_i(q)(p_j - p_i) + f_i$$  \hspace{1cm} (6)

where $f_i$ is a linear feedback for tracking particle $\hat{q}_i$ with state $\hat{x}_i$:

$$f_i = c_1(q_i - \hat{q}_i) - c_2(p_v - \hat{p}_v, \gamma); \hspace{1cm} c_1, c_2 > 0$$  \hspace{1cm} (7)

such that $n_{ij}$ is a subnormal vector connecting agent $i$ to agent $j$. Please refer to [14] for the definitions of $\phi_a$, the $\sigma$-norm $\|f\|_\sigma$, and smooth adjacency elements $a_i(q)$.

**Remark 1.** According to the flocking framework in [14], there exists a smooth potential function in explicit form

$$U_i(q) = \sum_{j \neq i} \psi_a(\|q_j - q_i\|) + \lambda \sum_{j \neq i} \|q_j - q_i\|^2$$  \hspace{1cm} (8)

with $\epsilon_q = 1/n \sum_{i=1}^n q_i$ such that $u_i$ can be stated as a distributed gradient-based control:

$$u_i = -\nabla_q U_i(q) + \sum_{j \in N_i} a_i(q)(p_j - p_i) + f_i.$$  \hspace{1cm} (9)

$\nabla_q$ denotes the partial derivative with respect to $q_i$.

Note that the state estimates generated by Algorithm 1 is directly used in equation (7) of Algorithm 2 for distributed mobility-control of the sensors. We refer to the combined Algorithms 1 and 2 as the cascade distributed estimation and control algorithm for collision-free distributed tracking of a
mobile target $\gamma$. The analysis of the this discrete-time coupled estimation and control algorithm is tremendously challenging and is one of our future research objectives.

In this paper, we seek to provide the stability analysis of the continuous-time version of this coupled distributed estimation and control algorithm.

IV. STABILITY ANALYSIS: COUPLED DISTRIBUTED ESTIMATION AND CONTROL ALGORITHMS

The formulation of our main analytical result as well as the following assumptions are inspired by our experimental observations and consistent collective behavior of a group of mobile sensors tracking two types of mobile targets: 1) a linear target and 2) a maneuverable nonlinear target called particle-in-the-box. Both models of the motion of targets will be discussed in detail in Section V. The notions of flocks, structural stability, and cohesion of flocks are used in the following proposition and defined in [14].

A flock is a connected network of dynamic agents. Flocking is the collective behavior of a network of dynamic agents with the objective to self-assemble and maintain a connected network in a collision-free manner. A flock is called cohesive if all the agents can be contained in a ball of finite radius.

Assumption 1. Assume there exists a finite time $T_1 > 0$ such that the proximity graph $G(q(t))$ becomes connected for all $t \geq T_1$.

The following definition clarifies that the Laplacian and algebraic connectivity of the networks used in flocking and KCF algorithms are not the same.

Definition 1. (Laplacian and $\lambda_2$ of the proximity networks in flocking vs. KCF) Let $a_{ij}(q)$ be the smooth adjacency elements of the proximity network of mobile agents with configuration $q = \text{col}(q_1, \ldots, q_n)$. We represent the adjacency matrix of flocking with $A_f(q) = [a_{ij}(q)]$ and its Laplacian and algebraic connectivity with $L_f$ and $\lambda_2^f = \lambda_2(L_f)$, respectively. The adjacency matrix $A_c = [a_{ij}^c(q)]$ of networked filters in KCF has 0-1 elements, i.e. $a_{ij}^c = 1$ if $a_{ij}(q) > 0$ and $a_{ij}^c = 0$, otherwise. Similarly, we denote the Laplacian and algebraic connectivity of the networked filters with $L_c(q)$ and $\lambda_2^c = \lambda_2(L_c)$, respectively.

Assumption 2. Assume there exist constant thresholds $\epsilon_1, \epsilon_2 \in (0, 1)$ such that the algebraic connectivity functions $\lambda_2^f(t) = \lambda_2(L_f(q(t)))$ and $\lambda_2^c(t) = \lambda_2(L_c(q(t)))$ along the trajectory of mobile agents cross the levels $\epsilon_1$ and $\epsilon_2$, respectively, at time $T_2 = T_2(\epsilon_1, \epsilon_2) > T_1$ and remain above those threshold values thereafter, i.e. $\lambda_2^f(t) \geq \epsilon_1$, $\lambda_2^c(t) \geq \epsilon$ for all $t \geq T_2$.

Assumption 3. The parameters $c_1, c_2 > 0$ in the tracking feedback $f^i$ of the flocking algorithm satisfy $c_1 < c_2 < 1$ and $c_2 > 1 - \epsilon_1$ where $\epsilon_1$ is defined in Assumption 2.

Here is our main theoretical result:

Proposition 1. Consider a network of $n$ mobile sensing agents with dynamics (1), the sensing model in (3), and the proximity graph $G(q)$ with the set of edges (4). Suppose that the agents apply the Kalman-Consensus filter in (5) to obtain $n$ estimates $\hat{x}_i$ of the state of a mobile target $\gamma$ with dynamics (2). These state estimates of the target determine the states of $n$ $\gamma$-agents $\gamma_i$. Suppose that every sensing agent $i$ tracks its associated $\gamma$-agent $\gamma_i$ by applying the flocking algorithm in (6). Let $\Sigma_c$ and $\Sigma_c$ be the collective dynamics of the $n$ networked estimators and mobility-controlled agents, respectively, and denote their cascade with $\Sigma$. Then, the following statements hold:

(i) $\Sigma$ can be separated into three subsystems that consist of the structural and translational dynamics of the group of mobile sensors in cascade with the error dynamics of the Kalman-Consensus filter.

(ii) Given Assumption 1, the agents form a cohesive flock in finite time.

(iii) Suppose that Assumptions 1 through 3 hold. Then, the solutions of the structural dynamics of the flock of mobile sensors are asymptotically stable.

(iv) Given the assumptions in part (iii), all estimators asymptotically reach a consensus on the state estimates of the target $\hat{x}_1 = \cdots = \hat{x}_n$ (for the error dynamics of KCF with zero noise).

The proof of proposition 1 is relatively lengthy; therefore, we present the proof in separate parts.

A. Proof of Part (i):

Let us first determine the error dynamics of the Kalman-Consensus filter in (5). The estimation error of sensor $i$ is defined as $\eta_i = x - \hat{x}_i$, thus error dynamics of (5) (without noise) is in the form:

$$\dot{\eta}_i = F_i \eta_i + \mu P_i \sum_{j \in N_i} (\eta_j - \eta_i)$$

with $F_i = A - K_i H_i$. Defining block diagonal matrices $F = \text{diag}[F_i]$ and $P = \text{diag}[P_i]$ and $\eta = \text{col}(\eta_i)$, one can rewrite the last equation as

$$\dot{\eta} = F \eta - \mu P \dot{L}_c \eta = F_c \eta$$

with $F_c = F - \mu P \dot{L}_c$. According to Theorem 1, the error dynamics $\dot{\eta} = F_c \eta$ is stable and has a quadratic Lyapunov function $V(\eta) = \eta^T P^{-1} \eta = \sum_i \eta_i^T P_i^{-1} \eta_i$.

The flocking dynamics of the agents can be written as

$$\begin{cases}
\dot{q}_i = p_i \\
\dot{p}_i = -\nabla_q U_\lambda(q) + \sum_{j \in N_i} a_{ij}(q)(p_j - p_i) - c_1(q_i - \hat{q}_i, \gamma) - c_2(p_i - \hat{p}_i, \gamma) 
\end{cases}$$

or

$$\begin{cases}
\dot{q}_i = p_i \\
\dot{p}_i = -\nabla_q U_\lambda(q) + \sum_{j \in N_i} a_{ij}(p_j - p_i) - c_1(q_i - q_j - \hat{q}_i, \gamma) - c_2(p_i - p_j + p_j - \hat{p}_i, \gamma) 
\end{cases}$$

After defining the block matrix $C = [c_1 I_{n1}, c_2 I_{n1}]$, one can express the last equation in a form with an input $\eta_i$:

$$\begin{cases}
\dot{q}_i = p_i \\
\dot{p}_i = -\nabla_q U_\lambda(q) + \sum_{j \in N_i} a_{ij}(q)(p_j - p_i) + f_i^T - C \eta
\end{cases}$$
with a linear tracking feedback

\[ f^T_i = -c_1(q_i - q_γ) - c_2(p_i - p_γ). \]

This enables us to express the dynamics of \( \Sigma \) as the cascade of its estimation and control subsystems \( \Sigma_e \) and \( \Sigma_c \):

\[
\begin{align*}
\Sigma_c : & \quad \dot{q}_i = p_i \\
& \quad \dot{p}_i = -\nabla U_\lambda(q) - D(q)p + f^T_i - \hat{C}\eta \\
\Sigma_e : & \quad \dot{\eta} = F_e\eta
\end{align*}
\] (12)

where \( D(q) = c_2I + \dot{L}_f(q) \) is a positive definite damping matrix, \( f = \text{col}\{f^T_i\} \), and \( \hat{C} = C \otimes I_n \) is a constant matrix. System (12) is the cascade normal form of estimation and control subsystems of a mobile sensor network in which its sensing agents apply the flocking algorithm for mobility control and the Kalman-Consensus filter for distributed tracking.

According to [14], since \( f^T_i \) is a linear feedback, the flocking dynamics \( \Sigma_c \) can be further decomposed as the cascade of structural and translational dynamics of particles. The position and velocity of the center of mass (CM) of the particles is given by

\[ q_c = \frac{1}{n} \sum q_i, \quad p_c = \frac{1}{n} \sum p_i. \]

Consider a moving frame centered at \( q_c \). Then, the position and velocity of agent \( i \) can be written as \( x_i = q_i - q_c \) and \( v_i = p_i - p_c \). We refer to the dynamics of the motion of the group of agents in the moving frame coordinates as structural dynamics. The structural and translational dynamics of \( \Sigma_c \) can be written as

\[
\begin{align*}
\Sigma_s : & \quad \dot{x} = v \\
& \quad \dot{v} = -\nabla U_\lambda(x) - D(x)v + \delta - \hat{C}\eta \\
\Sigma_t : & \quad \dot{q}_c = p_c \\
& \quad \dot{p}_c = -c_1(q_c - q_γ) - c_2(p_c - p_γ) + \delta
\end{align*}
\]

where the perturbation terms \( \delta = \text{col}\{\delta_i\} \) and \( \delta \) depend on the target estimation errors by the sensors and are defined as

\[
\begin{align*}
\delta_i = -c_1(q_i - q_γ_i) - c_2(p_i - p_γ_i) = -C\eta_i \\
\frac{1}{n} \sum \delta_i = -C\hat{\eta}; \quad \hat{\eta} = \frac{1}{n} \sum \eta_i = \frac{1}{n}(1_n^T\eta)
\end{align*}
\]

The normal form of \( \Sigma \) can be written as follows

\[
\begin{align*}
\Sigma_s : & \quad \dot{x} = v \\
& \quad \dot{v} = -\nabla U_\lambda(x) - D(x)v + \hat{\eta} + C(1_n^T\eta) \otimes 1_n \\
\Sigma_t : & \quad \dot{q}_c = p_c \\
& \quad \dot{p}_c = -c_1(q_c - q_γ) - c_2(p_c - p_γ) - C(1_n^T\eta) \\
\Sigma_e : & \quad \dot{\eta} = F_e\eta
\end{align*}
\]

The solutions of the structural dynamics in cascade with \( \Sigma_e \) is called cohesive for all \( t \geq 0 \) if the position of all agents remains in a ball of radius \( R_0 \) for \( t \geq 0 \). Note that this cascade nonlinear system is globally Lipschitz and all of its solutions are bounded for arbitrary initial conditions. The global Lipschitz property is a byproduct of the design of the smooth potential function \( U_\lambda(q) \) which has a globally bounded gradient. This implies that over the interval \([0, T]\) the solutions of the cascade system and therefore the position of all agent remain bounded. For all \( t \geq T \), the proximity graph \( G(q(t)) \) is connected and thus has a finite diameter \( d(t) \leq (n - 1) \) at any time \( t \). Define the diameter of the flock as

\[ d_{\text{max}}(t) = \max_{j \neq i} \|q_j(t) - q_i(t)\|, \quad t \geq T \]

Then, \( d_{\text{max}} = d(t)r \leq (n - 1)r/2 \) and by setting \( R_0 = (n - 1)r/2 \) the position of the agents remain cohesive for all \( t \geq T \) inside a ball of radius \( R_0 \).

To establish stability of the flock, we need to construct an energy-type Lyapunov function \( \varphi \) for the cascade of \( \Sigma_s \) and \( \Sigma_e \). Let \( H_\lambda(x, v) = U_\lambda(x) + \frac{1}{2}\|v\|^2 \) be the Hamiltonian of the unperturbed structural dynamics \( \Sigma_s \) and \( V(\eta) = \eta^T P^{-1} \eta \) be the Lyapunov function of \( \Sigma_e \). We propose the following Lyapunov function for the cascade nonlinear system \( (\Sigma_s, \Sigma_e) \):

\[ \varphi(x, v, \eta) = H_\lambda(x, v) + \frac{k}{2\mu} V(\eta) \] (13)

Before computing \( \varphi \), let us state a simple inequality. For an \( n \times m \) matrix \( M \) and two vectors \( x \in \mathbb{R}^n \) and \( y \in \mathbb{R}^m \), the following inequality holds:

\[ |x^TMy| \leq \frac{1}{2} (\|x\|^2 + \|My\|^2) \leq \frac{1}{2} (\|x\|^2 + \sigma_{\text{max}}^2(M)\|y\|^2) \]

In the special case of \( M = C = [c_1I_m, c_2I_m] \), we have

\[ |x^TCy| \leq \frac{1}{2} (\|x\|^2 + c_3\|y\|^2). \]

where \( c_3 = \max(c_1, c_2) \). By direct differentiation, we obtain

\[ \dot{\varphi} = \dot{H}_\lambda + \frac{k}{2\mu} \dot{V}(\eta). \]

From Theorem 1 and Assumptions 1 and 2, for all \( t \geq T_2 \), one gets

\[ \dot{V}(\eta) \leq -2\mu(\eta^T \dot{L}_e \eta) \leq -2\mu \lambda_2(\eta) \]

\[ \lambda_2 = \min_{t \geq T_2} \lambda_2(L_c(q(t))) \]

always exists based on Assumption 2.

Now, let us compute \( \dot{H}_\lambda(x, v, \eta) \). We have

\[ \dot{H}_\lambda = -v^T \dot{L}_f(x)v - c_2\|v\|^2 - \sum_i (v_i^T C\eta_i + v_i^T \delta). \]

Note that \( |v_i^T C\eta_i| \leq \frac{1}{2} (\|v_i\|^2 + c_3^2\|\eta\|^2) \) thus

\[ \sum_i |v_i^T C\eta_i| \leq \frac{1}{2} (\|v\|^2 + c_3^2\|\eta\|^2). \]

In addition, \( v_i^T \delta = \frac{1}{n} \sum_j v_i^T C\eta_j \). Hence

\[ |v_i^T \delta| \leq \frac{1}{2} \sum_j (\|v_i\|^2 + c_3^2\|\eta_j\|^2) = \frac{n}{2} \|v_i\|^2 + \frac{1}{2} c_3^2\|\eta\|^2 \]

and

\[ \sum_i |v_i^T \delta| \leq \frac{1}{2} (\|v\|^2 + c_3^2\|\eta\|^2). \]

Based on the above upper bounds, we get

\[ \dot{H}_\lambda \leq -v^T \dot{L}_f(x)v - c_2\|v\|^2 + \|v\|^2 + c_3^2\|\eta\|^2. \]
Given the fact that
\[ v^T \dot{L}_f(x)v \geq \lambda_2(L(x))\|v\|^2 \]
and setting \( \tilde{\lambda}_f^j = \min_{T-z} \lambda_2(L_f(x(t))) \), one concludes
\( \dot{\varphi} \leq (1 - c_2 - \tilde{\lambda}_f^j)\|v\|^2 + (c_2^2 - k\hat{\lambda}_f^2)\|\eta\|^2 < 0, \forall (v, \eta) \neq 0 \)
if the following two conditions hold:
\begin{equation}
\begin{aligned}
\tilde{\lambda}_f^j &> 1 - c_2 \\
\hat{\lambda}_f^2 &> c_2^2/k
\end{aligned}
\end{equation}
(14)

Given the definition of \( \tilde{\lambda}_f^j \) and \( \hat{\lambda}_f^2 \) and Assumption 2, we have \( \tilde{\lambda}_f^j = \epsilon_1 \) and \( \hat{\lambda}_f^2 = \epsilon_2 \). By choosing \( k \geq 1/\epsilon_2 \) and \( c_2 > 1 - \epsilon_1 \) (as in Assumption 3) both conditions will be satisfied. Thus
\( \dot{\varphi}(x, v, \eta) < 0, \forall (v, \eta) \neq 0 \)

Based on LaSalle’s invariance principle, for any set of initial conditions, the solutions of the cascade system \( (\Sigma_s, \Sigma_e) \) asymptotically converge to the largest invariant set in
\[ \mathcal{E} = \{(x, v, \eta) : \nabla U\lambda(x) = 0, v = 0, \eta = 0\} = \mathcal{E}_s \times \{0\} \]
where \( \mathcal{E}_s \) is the equilibria of the unperturbed structural dynamics. From the equilibria in \( \mathcal{E}_s \), only the local minima of \( U\lambda(x) \) are asymptotically stable.

The proof of part (iv) is a byproduct of the above stability analysis: the estimation errors \( \eta \) asymptotically vanish for all sensors and therefore all state estimates become the same.

**Remark 2.** If in addition to Assumptions 1 through 3, Conjectures 1 and 2 in [14] hold, then almost every solution of the structural dynamics of the flock asymptotically converges to a quasi \( \alpha \)-lattice. In all of our experimental results, we have observed finite-time self-assembly of quasi \( \alpha \)-lattices.

**V. EXPERIMENTAL RESULTS**

In this section, we apply our coupled distributed estimation and control algorithm—namely, KCF plus flocking—to two types of targets: 1) a target with a linear model which is a particle moving in \( \mathbb{R}^2 \), and 2) a maneuvering target with nonlinear dynamics. The later target remains in a rectangular region (box) for all time \( t \geq 0 \).

**A. Linear Target**

Consider a particle in \( \mathbb{R}^2 \) with a linear dynamics
\[ x(k + 1) = Ax(k) + Bw(k) \]
with
\[ A = \begin{bmatrix} I_2 & \epsilon I_2 \\ 0 & I_2 \end{bmatrix}, \quad B = \begin{bmatrix} (\epsilon^2/2)I_2 \\ \epsilon I_2 \end{bmatrix} \]
where \( \epsilon = 0.01 \) is the discretization step-size. The sensor makes noisy measurements of the position of the target, i.e.
\[ z_i(k) = H_i(k)x(k) + v_i(k); \quad H_i = [I_2 \ 0]. \]
The noise statistics for zero-mean Gaussian signals \( w(k) \) and \( v_i(k) \) are
\[ E[w(k)w(l)^T] = Q_k \delta_{kl}, \quad E[v_i(k)v_j(l)^T] = R_{kl} \delta_{kl} \delta_{ij} \]
where \( \delta_{kl} = 1 \) if \( k = l \) and \( \delta_{kl} = 0 \), otherwise. According to the model of information value in [8], the measurement error covariance matrix of sensor \( i \) is \( R_i = \frac{2}{f(\rho_i)}I_2 \) where \( f(\rho_i) \) is the information value function
\[ I_i = f(\rho_i) = 2I_0(a + b + (a - b) \frac{\rho_i - l}{\sqrt{1 + (\rho_i - l)^2}})^{-1} \]
(15)
where \( \rho_i = \|H_i(x) - q_i\| \), \( I_0 = 0.1 \), and \( a > b > 0 \). In our experiment, we use a mobile sensor network with \( n = 20 \) agents. The parameters of \( R_i \) are \( a = 8b \), \( b = 1 \), and \( l = 10d \). The interaction range of the agents in the flock is \( r = 1.2d \) and their desired inter-agent distance is \( d = 7 \). For the KCF algorithm, \( F_0 = 100I_4 \), \( x_0 \sim \mathcal{N}(0, \sigma^2 I_4) \) with \( \sigma = 60 \), and \( Q = 100I_2 \).

Fig. 1 shows the MSE of tracking error over 10 random runs, the average information value, and the algebraic connectivity plots during tracking. From Fig. 1 (c), one can readily verify that Assumptions 1 through 2 hold.

**B. Maneuvering Nonlinear Target: Particle-in-the-Box**

We also consider a maneuvering target with the following nonlinear dynamics:
\[ x(k + 1) = Ax(k)x(k) + Bw(k) \]
(16)
where \( x(k) = (q_1(k), p_1(k), q_2(k), p_2(k))^T \) denotes the state of the target at time \( k \). The target moves inside and outside of a square field \([-l, l]^2 \). Matrix \( A(x) \) is defined as
\[ A(x) = M(x) \otimes F_1 + (I_2 - M(x)) \otimes F_2 \]
\[ F_1 = \begin{bmatrix} 1 & \epsilon \\ 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1 - \epsilon c_1 & \epsilon c_2 \\ -\epsilon c_1 & 1 - \epsilon c_2 \end{bmatrix} \]
\[ M(x) = \begin{bmatrix} \mu(x_1) & 0 \\ 0 & \mu(x_3) \end{bmatrix} \].
where $F_1$ and $F_2$ determine the dynamics of the target inside and outside of the region, respectively, and $\mu(z)$ is a switching function taking 0-1 values defined by

$$
\mu(z) = \begin{cases} 
\frac{\sigma(a+z) + \sigma(a-z)}{2} & z \geq 0; \\
1 & z < 0 
\end{cases}
$$

$$
\sigma(z) = \begin{cases} 
1 & z \geq 0; \\
-1 & z < 0 
\end{cases}
$$

In addition, matrix $B$ is given by

$$
B = I_2 \otimes G, \quad G = \left[ \begin{array}{c} \epsilon^2 \sigma_0^2 / 2 \\ \epsilon \sigma_0 \end{array} \right].
$$

where $\epsilon = 0.03$ is the step-size, $\sigma_0 = 2$, $a = 45$, $l = 50$, $c_1 = 7.5$ and $c_2 = 10$ are the parameters of a PD controller, and the elements of $w(k)$ are normal zero-mean Gaussian noise with $Q = 100I_2$. The initial condition of the target is $x_0 \sim N(0, \sigma^2 I_4)$ with $\sigma = 2$ and $P_0 = 100I_2$. The parameters of the information value function in (15) are $I + \beta = 0.1$, $a = 10b$, $b = 1$, $l = 10d$ and $d = 7$. We consider a mobile sensor network with $n = 30$ nodes with a linear sensing model and

$$
H_i = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \end{array} \right].
$$

Fig. 2 illustrates the tracking estimation error, average information value, and the algebraic connectivity plots for the nonlinear target with snapshots shown in Fig. 3. Similarly, Assumptions 1 and 2 hold based on Fig. 2 (c).

VI. Conclusions

We introduced a theoretical framework for coupled distributed estimation and flocking-based control of mobile sensor networks for collaborative target tracking. The mobile sensing agents seek to improve the information value of their sensed data while avoiding inter-agent collisions. We demonstrated that the coupled dynamics of the combined distributed estimation and control algorithm has a separable cascade nonlinear normal form. Then, we provided the stability analysis of the structural dynamics of a flock with $n$ dedicated $\gamma$-agents in cascade with the error dynamics of the continuous-time KCF. Based on our experimental results, the discrete-time counterpart of the information-driven flocking algorithm is effectively applicable to tracking both a linear and a nonlinear maneuverable target.

REFERENCES


Fig. 3. Snapshots of a mobile sensor network tracking a maneuvering target with a flocking-based motion control algorithm. The target is marked by a red circle and the estimates are marked by green dots.