The story of how scientists developed in the last decade new techniques in computer simulations to finally understand one of the most important predictions of the century-old Einstein’s theory of gravity.

1. Prologue

In 1916, Albert Einstein published the result of his eleven-years research on the nature of gravity. One of the marvelous things of Einstein’s work was the discovery of a set of equations he believed encoded how we should describe the evolution of everything in the universe, from the motion of the planets through the creation and death of stars to the Big Bang. It is an elegant and beautiful theory. Despite these qualities, Einstein’s equation is very challenging to be solved. To this day, only a handful of solutions have been found. But thanks to computers and a whole new set of computational techniques developed in the last six years, we are finally discovering a previously hidden world of phenomena in the universe of black holes and neutron stars. Scientists are now able to describe the astrophysical dynamics of two black holes dancing with one another and the dramatic event of their collision. The blast of two black holes smashing on each other is brighter than all stars of the visible universe combined! This discovery was no small achievement, for it took more than forty years of trial and error in the implementation of elaborate computer programs. As an important byproduct of these developments, in 2005, the first complete prediction of the shape of the radiation emitted by the collision of two black holes was made, which will in turn serve for present and future telescopes to finally put to direct test one of the predictions of Einstein’s theory: that space and time can wrinkle back and forth in similar fashion to waves in a pond. In these next pages, we will examine Einstein’s theory, black holes and tell the tale of the challenges faced by physicists to put Einstein’s ideas inside a computer. Finally, we will learn about how astronomers intent to see in the near future telescopes the most violent astrophysical phenomena of the universe.
2. Einstein’s Legacy

We have an everyday experience with gravity: we call it the phenomena that makes stuff fall to the ground. But to understand gravity is another story. Where does it come from? Why is there gravity in the first place? These questions were tackled by Albert Einstein and his answers are the reason he is considered one of the great minds of the 20th century.

To explain how we understand gravity nowadays, let us start with the observation that when we measure distances on Earth, since the ancient times of the Egyptians, we do so by employing the traditional rules of Euclid’s geometry. That is to say that the path of smallest distance between two points in space is a straight line and the sum of the internal angles of a triangle is 180 degrees. Why is this true? Because these rules work: architects and engineers employ them everyday to design buildings and interior spaces. Nevertheless, mathematicians since Gauss have found that one can build mathematically-consistent geometries in which Euclid’s rules do not hold. A simple example is the geometry on the surface of a sphere. In this case, there is no way you can move between two points on a straight line.\footnote{You are not allowed to move inside the sphere, we are talking about the geometry on the surface.}

Another property of the geometry on the sphere is that if we draw a triangle (we connect three points on the sphere with arcs of circles), the sum of the internal angles is larger than 180 degrees. Thus, if we wanted to know whether or not the geometry of the real world was that of a sphere, all that we need is to test whether or not the sum of the angles of real-world triangles is 180°. A series of such experiments were indeed undertook by Gauss himself.

In 1916, Einstein pointed out that if we allow the geometry of the universe in space and in time to be free of Euclid’s ruling, we may achieve the extraordinary result of obtaining gravity as a consequence of the geometry itself. The basic idea is this: here on Earth we have seen that if an object is not being pushed by anything, it stays at rest or it moves in a straight line with a constant speed. An example of the latter case is when you roll a ball on a floor that is extremely slippery – the ball will just roll even after you are

Figure 1: Albert Einstein sitting alone at the Institute for Advanced Study. (From Life Magazine Archives, Nov. 1947, ©Alfred Eisenstaedt.)
no longer pushing it, and if there were no friction, it would roll indefinitely. However, if the rules of geometry were to change such that paths could no longer be straight lines, Einstein conjectured that the ball would follow the new curved paths in the modified geometry. For instance, if the geometry of the world were to be that of a sphere, objects would not move in straight lines when left alone, but in circles. Well, it just happens that the planets left alone in space move in circles around the Sun\textsuperscript{2}. Could it be that the reason planets have this path was the geometry on the scales of the solar system being different from Euclid’s? Einstein speculated that this was indeed the case.

But how does the geometry changes? Einstein eventually found an answer after nine or so years thinking about this. He approached this question by demanding a series of reasonable requirements on the equations that he wanted. First, the equations could not depend on the choice of the coordinates in the map of the geometry when localizing objects. This makes sense because a treasure map in the universe can tell us that a star A is at a 9 degrees longitude and 10 degrees latitude, or it can say that A is found somewhere in the straight line starting from Betelgeuse and ending at Venus; however, regardless of how we decide to describe the localization of the star, the distance from star A to the Earth should be the same. But if the shortest path from the Earth to the star A is an arc of a circle this will correspond to a different distance when compared to a straight line. In other words, once we pick a geometry, the distances should not depend on how we map it; but comparing different geometries, distances are allowed to change. So what can be used that is intrinsic of the geometry, but not of the map we choose? Mathematicians knew since the time of Gauss that the quantity that have this property is what they call the curvature of the geometry. Intuitively, the curvature of a geometry measures by how much two persons moving side by side with the same constant speed may drift apart or get closer. For instance, in a plane geometry, being at a constant speed makes you move in a straight line, hence two persons moving side by side on straight lines will maintain a constant distance. We then say that the plane has zero curvature. Whereas on a sphere, as they move along circles they also move away or closer to each other, so we say that the sphere has a non-zero curvature. Fig. 3 illustrates the idea. This nicely fits into our intuition that a a curved surface forces what is moving on top of it to follow non-straight paths.

So Einstein set up his equation to involve geometry only via its curvature. This however is only half of the story: what about the stars, planets and everything inside the

\textsuperscript{2}More precisely, the planets move around the Sun in ellipses, which are oval circles.
universe? Einstein reasoned that it was the matter of the universe that shaped the geometry, so he completed his equation by equating the curvature of geometry on one side to the energy content of what is in the universe in the other. A star with a much larger mass than the planets near it would curve the space around it to the point that “moving straight ahead” becomes moving in circles around the star. Empty space behaves as a flat geometry, but the presence of a massive star bends it in a manner as if space itself was a rubber sheet and the star was a heavy bowling ball placed on the top of it.

In science, when we say that we have a theory of something it means we have found an explanation of facts. Scientific theories are a set of fundamental principles, a logical framework, from which facts can be understood and predictions of new phenomena can be made. Einstein’s theory was put to several tests ever since it was proposed, and so far has passed all of them. One of the most remarkable examples of such tests was a discovery made in 1974 by the American astronomers Russell Hulse and Joseph Taylor Jr. They found two stars orbiting each other that were getting closer to one another at the exact same rate predicted by Einstein’s theory. They were awarded the Nobel Prize in Physics for this discovery in 1993.

But why are the stars found by Taylor and Hulse approaching one another? The reason is that in Einstein’s gravity, objects can make ripples in space that will move away from them just like waves in the surface of water that move away from a point where a rock felt in. The ripples in space are however of the geometry itself and for that are called gravitational waves. As a result, they carry away energy from the system. As the stars loose energy, they slowly get closer.

Even though we have seen from the Taylor-Hulse binary a strong case for the existence of gravitational waves, this phenomena predicted by Einstein’s theory have never been seen directly. In other words, we have not yet detected a wave-oscillation of gravity. It turns out that stars orbiting each other produce faint oscillations that are impossible...
to detect. If we would like to confirm Einstein’s theory searching for the missing gravitational waves, we must look elsewhere: black holes.

3. Black holes

The idea that every massive object in Nature exerts a gravitational attraction on others naturally led scientists to speculate if there could be something like a star so heavy that not even light could escape its gravitational pull. If one shoots objects upwards from the surface of the Earth with increasing speed, eventually the object will be moving so fast that it will be able to escape the gravitational pull of the Earth and move indefinitely away into space. The velocity needed for an object to escape the gravitational pull of a planet or a star is called the escape velocity. Following Newtonian physics, one can conclude that if all the atoms of the Sun were compressed in a radius 4 million times smaller, then its escape velocity would be higher than the speed of light. The result would be a massive black void in space, since light would not be able to escape the Sun! The star would then be a black hole. The effect goes beyond the darkness of visible light: radio signals, microwaves and so forth, also propagate at the speed of light and hence would be trapped in the surface of the black hole. A person inside a black hole would not be able to send any signals of his or her existence to the outside world. He or she would also not be capable of escaping: since nothing moves faster than light in the known universe, nothing can escape the gravitational force of this object. Everything that falls into a black hole is prevented from gravity from ever leaving.

That such an object could exist was thought to be a mild amusement by many scientists for a long time. The story started to become more serious when physicists unveiled
the mechanism that keeps two objects from occupying the same place, the exclusion principle. In general, it was thought, electrons in atoms will relax to the smallest orbit allowed near the protons. However, if all electrons could do that, then their electric repulsion resulting from all sitting in the same orbit would make many-electrons atoms unstable, in gross disagreement with the stability of atoms such as iron. To circumvent this issue, the Austrian physicist Wolfgang Pauli postulated that two electrons in the universe cannot have the same state of motion. An electron can be moving around a proton in the closest orbit allowed and then it can spin around its own axis. But an electron can only spin clockwise or counter-clockwise, therefore only two electrons can be placed in a atom moving both in the path closest to the proton (each spinning in opposite directions). A third electron added to a two-electron atom then has to move in the next-to-closest to the proton orbit in order to satisfy the exclusion principle.

Nonetheless, the exclusion principle cannot be an infinite repulsive force. This was first realized in 1928 by Subrahmanyan Chandrasekhar, an Indian at the time graduate student at Cambridge University in England. He pointed out that since nothing is allowed to move faster than the speed of light, the repulsion force of the exclusion principle should not impart a speed to the electrons higher than the speed of light. Chandrasekhar showed that under this assumption, a star that was heavier than about one and a half times the mass of the Sun would suffer a gravitational force created by its own mass bigger than the maximum force of the exclusion principle. This mass of one and a half times the mass of the Sun is now called the Chandrasekhar limit. While the star is burning hydrogen nuclei to form heavier elements, the energy released by nuclear fusion keeps it from being smashed by gravity. However, as soon as all the nuclear combustible is used, gravity will act alone against the exclusion principle and will win, resulting in an object that can shrink to a point! But if a star with a mass bigger than the Chandrasekhar limit shrinks to a radius of about 4.5 km, the escape velocity near its surface will be bigger than the speed of light. A black hole then should form.

Since the work of Chandrasekhar, the question of whether or not black holes existed in Nature remained controversial until the mid 1990s. Most scientists, including myself, became convinced of the existence of black holes after seeing precise measurements on the motion of stars and interstellar gas. The first very compelling case was a discovery in 1994 made with the Very Long Baseline Array radio telescope. The astronomers William Watson and Bradley Wallin at the Department of Physics of the University of Illinois at Urbana-Champaign, showed that a gas cloud in a nearby galaxy, known as Messier 106, was moving in an elliptical orbit around a small black void. Because the orbit of the cloud was so precisely elliptical, it could only be that it was moving around an object that was about the size of a star or smaller. But given the mass of the cloud and its orbit, the mass of the object that was exerting the gravitational force was calculated to be about 40 million times the mass of the Sun! Such an object is safely above the Chandrasekhar limit, it is roughly spherical and it does not emit light. Based on the atomic physics we can test on Earth laboratories, the conclusion is inevitable that Messier 106 has a black hole in its core.
Figure 5: A photograph of Messier 106. The motion of a gas cloud in the center of this galaxy was the first strong evidence for the existence of black holes (© Sloan Digital Sky Survey).
The most direct evidence of a black hole to this day is the observation of the center of the Milky Way. It was discovered in 1974 by the astronomers Bruce Balick (University of California, Santa Cruz) and Robert L. Brown (National Radio Astronomy Observatory) that there existed in the center of our galaxy some astrophysical object that was emitting a lot of radio signals. The object became known as Sagittarius A* (pronounced “A-star”), or Sgr A* for short. Improved resolution of images of this region showed that Sgr A* was black, not emitting any light, but surrounded by many stars. From 1992 to 2002, a group led by Rainer Schödel of the Max Planck Institute for Extraterrestrial Physics in Garching, Germany, made observations of a star’s orbit around Sagittarius A*. In a paper published in *Nature* in October 17th 2002, they showed that the star was in an elliptical orbit (Fig. 7). The shape of the orbit implied that Sgr A* 1) has a mass of about 4 million times the mass of the Sun and 2) has a radius smaller than 26 thousand times the radius of the Sun. The escape velocity of such an object is bigger than the speed of light, which indicates it must be a black hole.

There is also other sources of evidence for black holes, which lead us to believe that there exists at least one in the center of every galaxy. Some of this evidence include the following. The existence of intense bright centers of many galaxies, which are called Active Galactic Nuclei (AGN), require the existence of a small compact, very heavy object that drags electrons from the nearby gases to create very intense light emissions. The best candidates for many of these AGNs are black holes. There are also systems in which a star is seen to be orbiting another object and emitting a great deal of X rays. The best known example of such is the Cygnus X-1, which now we believe to be a star orbiting a black hole in the Cygnus constellation. The reason why the companion of the Cygnus X-1 star is likely to be a black hole, is because the matter of the star is detected to be falling into the companion at a speed very close to that of light. No known star can have such a dragging effect, but a black hole can. Finally, in 2005, Warren Brown and collaborators at Harvard University discovered a star in the Milky Way with a speed of 850 km/s, which is higher than the escape velocity of our galaxy. This means that the star was literally ejected from the galaxy by a giant astrophysical slingshot. The existence of these slingshots was predicted years before in 1988 by the theoretical physicist Jack Hills at Los Alamos Laboratory. He showed that when two stars bound together passes by a black hole, one of them can be captured falling into the black hole, while the second suffers a change in its velocity that can be as high as 4,000 km/s. This phenomena is analogous to what happens when an ice skater holding a ball decides to throw the ball: he or she moves in the opposite direction of the pitch. Now, imagine if you are holding a ball and a supermassive black hole engulfs it and you can see why stars are catapulted out of the galaxy when they loose a companion to a black hole!
Figure 6: A photograph of the center of our galaxy made with the W. M. Keck Telescope. There is an object labeled Sgr A* that does not emit light but is surrounded by many stars. It is the supermassive black hole that lives in the center of the Milky Way (© W. M. Keck Observatory / UCLA).

Figure 7: The measured orbit of one of the stars that live in the center of the Milk Way. The black dots with crosses are the positions of the star. These are labeled with the year in which the star was found to be there. The location of the central black hole is indicated. (R. Schödel et al., Nature 419, 694, 17 Oct. 2002)
4. It is hard to simulate a black hole

Since the mid 90s when evidence for black holes became strong and we learned that all galaxies host at least one, astrophysicists started to focus more in understanding what these objects can do. Can black holes influence the formation of stars in galaxies? Does the emission of light by particles falling into gravity’s trap demonstrate that these objects are truly described by Einstein’s theory? One important line of research is this: what happens when two black holes collide?

It was always expected that when two black holes fuse together they release a giant amount of energy. Since these are the astrophysical objects with the strongest gravitational force possible, their interaction was expected to produce a very large amount of gravitational waves, those oscillatory disturbances of space believed to be the reason why the Taylor-Hulse binary has an orbital decay. The direct detection of an oscillation in space became a major scientific challenge. Also during the 90s, a joint collaboration of scientists from Caltech and MIT was succeeding in implementing an experiment called LIGO to detect gravitational waves. However, deriving the exact shape of a gravitational wave from Einstein’s theory turned out to be a very hard problem.

A general solution of Einstein’s equations of gravity for two massive bodies to this day is not available. One of the reasons is because even though a solution of one isolated black hole is already known, Einstein’s equation do not lend themselves to a simple addition property: we cannot add two black hole solutions together to obtain another solution to Einstein’s equations. This means that two black holes close to one another curve the space around them in a way that is dramatically different than having just one. Both theoretical physicists and mathematicians have spent a great deal of time trying to figure out ways to solve Einstein’s equation but no general method has emerged.

With the advent of computers, physicists turned to the possibility of programming one to solve Einstein’s equations. The first attempt in this direction was made in 1964 by Susan Hahn from IBM in New York City, and Richard Lindquist from Adelphi University in Long Island, New York. They ran a program in a IBM 7090 mainframe to solve Einstein’s equations starting with an approximate solution of two black holes very far away but moving towards a head-on collision. When the holes are far apart, one can approximate the space to be that of empty space slightly curved by the two holes. As the two holes approach one another, space becomes very curved near them and can no longer be described as empty. What exactly is the curvature of the space was the task Hahn and Lindquist programmed the computer to calculate. They attempted to start with a simplified version of the problem by restricting the holes to move in a two-dimensional plane instead of three space dimensions.

The problem of Hahn and Lindquist can be easily understood with an analogy with digital filming. The real world has an infinite number of points in space, but a digital camera recreates this world splitting it in a grid of points, the pixels. The camera then records the color of light and intensity at each of these points of the grid. If there is a large number of points, then at some distance from the picture it will look very smooth to our eyes, instead of a collection of discontinuous colored points. We then say that the
grid of pixels approximates the real world for the purposes of a certain picture size. To create a movie, the camera records a series of instantaneous photographs at a constant rate. For instance, standard movies are displayed in computers with 24 to 30 pictures per second.

Similarly, Hahn and Lindquist split the 2-dimensional space in which the black holes were moving into almost 8,000 pixels separated by a distance equal to one hundredth of the radius of the holes. They were interested in creating the film showing the evolution of the two holes according to Einstein’s equations. This is done in steps in time like the frames per second of a digital movie. In the first frame, at each pixel they used Einstein’s equations to calculate the space curvature. With the curvature of the space from the first frame, one can calculate what is the force of gravity acting on each hole, which is then used to calculate where the holes will move to. The holes then move to their new positions predicted by Einstein’s equations in the second frame, and the process of computing the curvature is repeated, which then gives the position of the holes for the third frame. This then goes on as the full film is created. After three hours of calculation by the IBM computer, the black holes were a distance apart equal to ten times their radius at which point the computer crashed: numerical errors were dominating the computation, the numbers were out of control. After the work of Hahn and Lindquist, other physicists during the 1970’s attempted to solve Einstein’s equations in a computer to no avail: even as computers were getting better, they were also always breaking down when faced with Einstein’s theory!

As the Caltech & MIT LIGO collaboration efforts showed during the mid 90’s that measuring gravitational waves could be feasible, the National Science Foundation established in 1997 a Grand Challenge grant to support solving Einstein’s equations in a computer. The understanding of why the computers were crashing finally came with a ground-breaking work in 2002 by the physicists Gioel Calabrese, Jorge Pullin, Olivier Sarbach, and Manuel Tiglio, then at the Louisiana State University. They proved that Einstein’s equations once written in a computer could be strongly chaotic depending on the choice of the grid used. The chaotic behavior means that small random numerical errors, such as rounding errors, were being amplified by Einstein’s equations leading the computers to crash. They showed that the amplification was independent of the resolution used; that meant that no matter how small the distance between points of the grid was, any numerical error would grow to dominate the whole calculation after a few steps — in the digital movie analogy, meaning after a few frames.

We can understand this discovery in the analogy of the movie camera. Suppose that we are trying to shoot a film of a green screen that does not move. The camera will capture the first frame, which will be mostly green. There are random errors in the capture procedure, for instance, one pixel in the picture may come out red. In digital filming and photography this is called “noise”. Einstein’s equations are such that every single small

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3More precisely, they showed that in certain coordinates, several numerical methods are not guaranteed to converge to a solution Einstein’s equations, regardless of resolution.
red dot in a otherwise green grid may be amplified to a giant red smudge until the picture becomes entirely red!

But since the problem was related to the way in which the grid was constructed, choosing different grids would get rid of the instabilities. What this means is that it was necessary to slice space in an astute manner to solve the problem. As an example, if the pixels are all at the same distance from one another, this is a rectangular grid, in which we move from one pixel to another in space along a line. But one can also move along a circle or an ellipse, giving rise to the so-called circular polar and elliptical grids (Fig. 8). After the discovery of Calabrese and his group, physicists sought alternative ways in splitting space to obtain a realization of Einstein’s equations free of chaos. This finally lead in less than three years later to the first successful computer simulation free of numerical instabilities. The solution is more complicated than a simple static rectangular, elliptical or circular grid: the grid actually needs to be continuously changed as the simulation progresses.

5. Refinement and Harmony

In 1999, a young graduate student, Frans Pretorius of the University of British Columbia, started his Ph.D. work on the topic of solving Einstein’s equations in a computer. That year was in the midst of increasing interest in the solution to this problem. Since at that moment the discovery of Calabrese and collaborators was still in the future, Pretorius and his advisor, Matthew Choptuik, decided to pursue the idea of increasing the resolution of the space pixelation near the black holes, hoping this could purge computer crashes. The technique was already known in simulations of fluids used in problems of aerodynamics and engineering, but its implementation in gravitational physics was still lacking. They developed a computer code in which the density of pixels across the grid continuously changes with time to make sure that at each instant the regions where the curvature of space is more pronounced the resolution is higher than everywhere else. The curvature is bigger near the black holes, so the code continuously changes the resolution.
of the grid keeping it high near the holes. This is known as the adaptive mesh refinement. For his thesis work, Pretorius was awarded in 2003 the Metropolis Prize for best dissertation in Computational Physics from the American Physical Society. Pretorius went on to the California Institute of Technology in Pasadena as a post-doctoral fellow to continue this research. After the discovery of the chaotic nature of Einstein's equation dependent on the choice of grid, Pretorius started to study how to select the best pixelation to avoid the issue.

Pretorius then came across an interesting work from October of 2001 by David Garfinkle of Oakland University in Michigan. Not knowing of the detailed analysis of Calabrese and collaborators, Garfinkle reasoned following intuition that if the coordinates of the grid itself were to follow a wave-like equation then numerical instabilities could be converted into oscillations that would not grow in time. However, he knew already the simplest approach could not work because a simple wave-like equation for the coordinates could itself have numerical instabilities. Hence he proposed a generalization of the wave equation free of such problems. Garfinkle then wrote a computer program that solved Einstein's equations in the presence of a single heavy blob of mass and saw no numerical instabilities coming from the grid. This became known as the generalized harmonic coordinates. Could it be that Garfinkle's method was the guarantee for the complete avoidance of chaotic behavior of Einstein's equations?

The answer finally came three years later in December 23rd, 2004, when Pretorius published a paper showing that no chaotic behavior was generated by the grid of a generalized harmonic coordinates in the case of black holes. He then changed the wave equation of the coordinates a little bit to improve the accuracy of the computer simulation. This suggested that the problem of numerical instabilities could be after all solved with a slightly modified version of Garfinkle's method. In the following months, Pretorius worked in writing a computer program to implement this new idea with his previous methods of adaptive mesh refinement. In the 4th of July of 2005, he stunned the scientific community with the very first 3D simulation of two black holes moving together, then orbiting each other several times until the emission of radiation brought both to a complete collision. He ran his simulation in a cluster composed of sixty-two Pentium III (850MHz) Intel processors with 511MB of RAM each. At last, the problem of coding Einstein's equations was solved.

From Pretorius simulation, we learned that about 5% of the total initial mass of the
two holes is converted by the fusion process into energy of the gravitational waves. The amount of energy released per second in gravitational waves was one million billion billion (10^{24}) times the energy per second in light released by the Sun. In the observable universe we can see roughly a 100 billion galaxies, each with around one to ten billion stars, which means that we can observe a thousand billion billion stars (10^{21}). Our Sun is known to be the average star type, thus a single pair of black holes merging is an event brighter in gravitational waves than all the stars of the visible universe are in light combined. It is the single most explosive event in the universe known so far. If two black holes with the mass of Sgr A* merge, they will burst into gravitational waves with this brightness during roughly 16 minutes. The whole process is depicted in Fig. 10, which shows the result of an actual computer simulation.
Figure 10: Complete simulation of two black holes orbiting one another until they merge into a single hole. Black holes are shown as grey spheres, while the amplitude of the gravitational wave disturbance on space is shown in color from strongest green to weakest blue. Empty undisturbed space is shown as a dark blue. Top row: the black holes are approaching in a spiral motion until the curvature of the space between them is no longer the same as empty space. In the last picture of the top row, the space near the holes is already highly curved and oscillatory. Middle row: as the two holes spiral down to merge, they emit several very strong bursts of radiation, signaling that the merger is about to occur. Bottom row: the merger occurs and the two holes smoothly amalgamate into a single hole, exploding in gravitational waves after which a single remnant rotating black hole is seen. Full video available at http://numrel.aei.mpg.de/images (©C. Reisswig, L. Rezzolla (simulation), M. Koppitz (visualization), Max Planck Institute for Gravitational Physics, Potsdam, Germany).
6. Was Einstein right, after all?

Pretorius work made available a detailed shape of the gravitational wave emitted by coalescing black holes. This in turn will be used directly in searches of gravitational waves. At this point, you may naturally ask: but will black holes collide in nature? The answer is yes. This is because astronomers have observed that galaxies very routinely meld together. It is seen that most galaxies now have merged at least once with another galaxy. Indeed, our own galaxy is right now in the process of merging with its satellite companion galaxies, the two Magellanic Clouds.

When galaxies merge together, the central supermassive black holes of each will come near each other and form a binary system. Dissipation caused mostly by the interstellar gas around them will bring both together until they collide. With the present rate of galactic mergers, it is expected that between three to two hundred black holes binaries merge every year. If at least one of this can be seen from Earth, we will have finally detected the oscillations of space and time predicted by Einstein.

Nevertheless, gravity is very weak when it is not related to heavy stuff like a billion solar-mass black hole. Even though our bodies are kept in contact with the Earth due to the gravitational pull of the planet, we can easily raise our arms and small objects against the work of gravity. How strong is the gravitational oscillation caused by a black hole merger nearby on a terrestrial object? Well, not much. If a nearby merger occurs, a typical terrestrial object will move a billion billion fraction of a meter ($10^{-16}$ cm). This is about one million times smaller than the radius of the hydrogen atom! Measuring this type of small deviation is an outstanding technological challenge. It seems utterly absurd to even speak of measuring the motion of a macroscopic object to a fraction of the radius of an atom, but light, unlike matter, is not granulated in space and thus small deviations in light signals can be in principle seen to arbitrary small length scales. This is the concept of the Laser Interferometer Gravitational Wave Observatory, or LIGO, a joint collaboration started in the late 80’s between physicists Reiner Weiss, from MIT, and Ronald Drever and Kip Thorne from Caltech.

The concept behind LIGO is called a light interferometer. In a L-shaped structure, two mirrors are placed in each leg of the L, and a semi-transparent material is placed in the vertex (cf. Fig. 11). Laser light is shot through the vertex and half of it is transmitted straight to the mirror in front while the other half is reflected towards the other mirror. The mirrors reflect light back through the same path and the re-combined light ray is then seen on an adequate camera (detector). The phenomena of light is an electromagnetic...
wave, which means it is an wave-like oscillation of electricity and magnetism. The electric oscillation is picked up by the camera and its pattern can then be seen in a computer. When left alone, the system will have the natural oscillation frequency of the light source used. But when a gravitational wave hits the Earth, space itself will contract and expand at the frequency of the gravitational wave making the path of light oscillate. This is a new oscillation on the top of the natural one of the whole system and causes the flashes of light detected in the camera to have an extra swinging. This extra fluctuation is what can be identified as a signal of the gravitational wave. Still, measuring a small deviation in light-path of $10^{-16}$ cm is a great challenge.

The LIGO experiment started with the construction in 2000 of two interferometers, one in Livingston, Louisiana, and the other in Hanford Nuclear Reservation near Richland, Washington. The project is funded by the National Science Foundation. The basic apparatus was concluded shortly after the construction started and the first scientific tests of the machine were made in 2002. In June 2006, the LIGO collaboration established that the instrument was capable of measuring deviations as small as $10^{-16}$ cm in the path of light, which was their goal. In the fall of 2011, the instruments were closed for the next stage of updates which includes several important improvements, such as a stronger laser beam and better control of seismic effects. These refinements aim in providing the tiny sensitivity of $10^{-16}$ cm to a wide range of gravitational wave frequencies. The new band
of sensitivity is expected to allow the detection of nearby black hole mergers. Both LIGO sites are scheduled to be ready for action in 2014. If the upgrade goes well, and if the experiment sensitivity is confirmed, at least one to two additional years will be necessary to confirm whether or not gravitational waves were found.

7. Outlook and Conclusion

It is now clear that the toughest obstacles in solving Einstein’s theory of gravity in a computer have been settled. Nevertheless, the path of exploration has just begun. Since the work of Pretorius, calculations have now been extended in several directions for astrophysics. The complete evolution of the orbit of one black hole and a star, or of two stars is one example. Another one is the complete simulation of the evolution of nearby intergalactic gas during a black hole collision or when two neutron stars merge. The latter is very important to understand the astrophysical phenomena of gamma ray bursts, and are currently speculated to be one of the sources of these phenomena, though the evidence is far from conclusive.

Simulations of Einstein’s theory in a computer can lead in the future to more than detailed description of astrophysics. Physicists currently believe that many different phenomena in Nature, such as nuclear reactions under very high pressure or superconductivity, can be described by mathematics identical to gravity. In the past, complicated gravitational models of these phenomena were out of reach, but now they are being explored in computer simulations.

Still, the greatest achievement of numerical relativity for now is the provision of the wave-forms for LIGO and other gravitational wave detectors, which will put a new test to Einstein’s theory of gravity and perhaps open the window for gravitational-wave Astronomy.

8. Further reading


